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Robust controller design for flexible
structures.

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I. Problem Definition.

Consider the problem of control of a beam as shown in Fig. 1. The beam is moving in the x-y plane. It extends from x=0 to x=L. The left end at x=0 is clamped to an actuator which moves the beam along the y-axis. The control input is the force u(t) in y direction. While moving, the beam may vibrate. Let z(t) denote the displacement of the left from y=0 and w(t,x) the displacement of the beam from the line y=z(t) at position x and time t. Suppose a position sensor is place on the beam and the sensing output is v(t, x_0)=z(t) + w(t,x_0), where 0<x_0<L is the sensor location. We are interested in the case when the flexure w(t,x) of the beam is significant. The problem is to synthesize a feedback control law which moves the beam from one position to another in a stable manner.

It is well known [1] that when the sensor and the actuator are colocated a simple lead compensator suffices to produce a stable design. This result holds even when the beam dynamics are consided as a system with infinite zero-damping modes, and can be shown using root locus arguement [2]. This stabilization method may break down, however, when there is a positional gap between the sensor and actuator. In this case the classical compensation techniques are no longer effective. Time-domain optimization approaches based on state-space models have been applied to this problem, see for example [3]. In this article we present a case study of noncolocated beam control problem using frequency-domain optimization method proposed by Professor Kwakernaak [4, 5]. We emphasize the choice of the weighting functions in the cost function, and the search method which always leads to stable designs.

II. Dynamic Models.

Assume the beam has a constant mass desity ρ, and a constant bending rigidity EI. The dynamics of the beam, under the assumption of negligible shearing, satisfy the equations

$$\rho L (d^2 z/dt^2) + \rho \quad (\partial^2 w/\partial t^2) \, dx = u(t)$$

$$\rho (d^2 z/dt^2) + \rho (\partial^2 w/\partial t^2) + EI (\partial^4 w/\partial x^4) = 0 \qquad (1)$$

$$y(t) = v(t, x_0)$$

with boundary conditions

$$w(t, 0) = w_x(t, 0) = w_{xx}(t, L) = w_{xxx}(t, L) = 0 \tag{2}$$

Let ϕ_i be the eigenmode shapes of the beam. We discretize the dynamics by the decomposition

$$w(t, x) = \sum_{i=1}^{\infty} q_i(t)\, \phi_i(x) \tag{3}$$

Let $\xi = (z, q_1, q_2, \ldots)^T$. We obtain an equivalent model of infinite dimension as

$$M\ddot{\xi} + K\xi = F u$$
$$y = C\xi \tag{4}$$

where

$$M = \begin{bmatrix} \rho L & \rho \int_0^L \phi_i\, dx \\ \rho \int_0^L \phi_i\, dx & \rho \int_0^L \phi_i\, \phi_j\, dx \end{bmatrix} \qquad K = \begin{bmatrix} 0 & 0 \\ 0 & EI \int_0^L \frac{\partial^2 \phi_i}{\partial x^2} \frac{\partial^2 \phi_j}{\partial x^2}\, dx \end{bmatrix}$$

$$F = (1\ 0\ 0\ \cdots\ )^T, \text{ and } \quad C = (1\ \phi_1(x_0)\ \phi_2(x_0)\ \cdots\ ).$$

Suppose the sensor is colocated with the actuator at x_0, then the output matrix becomes

$$C = (1\ 0\ 0\ \cdots\ ).$$

This input-output relation gives rise to a well-known pattern (see [1, 6]) of poles and zeros as shown in Fig. 2. A simple proof is given on the basis of root locus arguement in [2].

Notice that in this modeling the beam is assumed to have no structural damping, which results in all the poles and zeros lying on the imaginary axis. In reality this is of course not the case. Typically flexibility modes of very high frequency tend to have good damping. For

stabilizing the system described above, it is therefore possible to first truncate the high-frequency modes and concentrate on a finite number of modes with zero damping.

Consider a finite-dimensional model after truncation. Suppose the position sensor moves away from the location of the actuator. It can be shown that the zeros will move upward along the imaginary axis. Beginning from the highest mode the zeros will cross the poles while the poles remain at the same locations. At some point the zeros will break into complex patterns and the system has nonminimum phase zeros. This observation is very important, since nonminimum phase zeros impose severe constraints to control synthesis. In Fig. 3. the zero movement due to various sensor placements is shown with a three meter auminum beam.

III. Lead Compensation.

It has been observed (see [1]) that a lead compensator suffices to stabilize every mode of the system in (4) when the sensor and the actuator are colocated. That is, the following theorem holds.

THEOREM 1. Consider an open-loop system

$$G(s) = K_1 \prod_{i=1}^{n} (s - z_i) / [s^2 \prod_{i=1}^{n} (s - p_i)] \tag{6}$$

where $K_1 > 0$, and $0 < z_i < p_i < z_{i+1}$ for all i. Let H(s) be a lead compensator

$$H(s) = K_2 (s/z + 1)/(s/p + 1), \quad z<p. \tag{7}$$

Then there exists a positive real number K^* such that for any K in the interval $(0, K^*)$ the closed-lop system

$$M(s) = G / (1 + GH) \tag{8}$$

is asymptotically stable.

A sketch of the proof of this theorem is illustrated in Fig. 4. The departure angle θ_i of the root locus from the i-th mode satisfies the equation

$$\theta_i = \theta_p - \theta_z + \pi / 2 \qquad\qquad (9)$$

where θ_p and θ_z are the phases of the pole and the zero of the lead compensator. It is clear that for a small gain the system is stable. Now suppose for a certain mode the order of the pole and the zero is reversed as shown in Fig. 5, which corresponds to a noncolocated system. With a lead compensator the departure angle from this pole can be computed as

$$\qquad\qquad (10)$$

It is clear that the system is unstable for small gains.

IV. Kwakernaak's Minimax Approach.

As shown in Section III, the classical compensation techniques are not effective in control synthesis for noncolocated flexible beam with zero-damping modes. Our goal in this section is to apply the minimax approach proposed by Professor Kwakernaak [4] to this problem. We present only the procedural aspects of the design. For details of the supporting theorems the reader is referred to the paper [4].

Consider again the truncated system (6) of the flexible beam. Let ψ and ϕ be respectively the zero polynomial and the pole polynomial. Consider a unity feedback configuration and let $H(s) = \zeta(s)/\sigma(s)$ be a controller where ζ and σ are polynomials. The sensitivity function S and its complement T are computed as

$$S(s) = (1 + G H)^{-1} = \phi \sigma / \chi ; \quad T(s) = 1 - S = \psi \zeta / \chi \qquad (11)$$

where $\chi = \phi \sigma + \psi \zeta$ is the closed-loop characteristic polymonial. The objective of Kwakernaak's minimax approach is to find a controller $H(s)$ which minimizes the cost function

$$\text{Sup}_\omega \left(\, |V(j\omega) \, S(j\omega)|^2 + |W(j\omega) \, T(j\omega)|^2 \, \right) \qquad (12)$$

There $V(s)$ and $W(s)$ are weighting functions. We denote

$$Z(s) = V(s) \, V^*(s) \, S(s) \, S^*(s) + W(s) \, W^*(s) \, T(s) \, T^*(s) \qquad (13)$$

where $V^*(s) = V(-s)$. The objective is then to minimize $\text{Sup}_\omega \, Z(j\omega)$. It is shown in [4] that if an optimizing solution exists, it is unique and it satisfies

$$Z(s) = \lambda^2 \qquad (14)$$

For the beam control problem we let the weighting functions be

$$V(s) = \alpha_1 / \beta_1 = (1 + \tau_1 s^2) / (\tau^2 \phi)$$

$$W(s) = \alpha_2 / \beta_2 = (\tau_2 s^{n+2}) / \psi \qquad (15)$$

where τ_1, τ_2 are positive constants to be chosen as design parameters. These weightings satisfy the conditions:

(1) $\deg(\alpha_1) < \deg(\phi)$, $\deg(\alpha_2) = \deg(\psi) + $ pole excess.

(2) α_1, α_2, and β_1 have their roots in the closed LHP. The polynomials α_1 and β_1 have no common roots.

(3) The polynomial $\gamma = \alpha_1 \alpha_1^* \beta_2 \beta_2^* + \alpha_2 \alpha_2^* \beta_1 \beta_1^*$ has no roots on the $j\omega$-axis.

Under these conditions it can be shown that the minimax problem has a solution. With these weightings and $H(s) = \zeta / \sigma$, the condition (14) can be expressed as

$$\alpha_1 \, \alpha_1^* \, \sigma \, \sigma^* + \alpha_2 \, \alpha_2^* \, \zeta \, \zeta^* = \lambda^2 \, \chi \, \chi^* \qquad (16)$$

Let

$$\eta = \alpha_1 \, \alpha_1^* \, \alpha_2 \, \alpha_2^*$$

Consider the polynomial $\gamma - \eta/\lambda^2$ with λ as a parameter. Let λ_0 be the first value for which the polynomial loses degree or assumes a root on the imaginary axis. It can be computed that

$$\lambda_0 = (\tau_1^2 \, \tau_2^4) / (\tau_1^2 + \tau_1^4 \, \tau_2^4). \tag{17}$$

Now let H(s) be a reduced-order compensator for which

$$\zeta(s) = \sum_{i=1}^{n} \zeta_i \, s^{n-i}, \quad \sigma(s) = s^n + \sum_{i=1}^{n-1} \sigma_i \, s^{n-i} \tag{18}$$

With a particular λ the equation (16) gives a solution $(\zeta_\lambda, \sigma_\lambda)$. However, a solution as such may be unstable. It is shown in [4] that there exists a $\lambda > \lambda_0$, which yields an optimal stablilizing compensator. To facilitate searching for stable solution we take the following approach. Notice that the equation defining the closed-loop characteristic polynomial

$$\sigma \, \phi + \zeta \, \psi = \chi = s^{2n-1} + \sum_{i=1}^{2n-1} d_i \, s^{2n-i-1} \tag{19}$$

can be put into the form

$$S(\phi, \psi) \, C = D \tag{20}$$

where

$$S(\phi, \psi) = \begin{pmatrix}
1 & 0 & \ldots & 0 & b_1 & 0 & \ldots & 0 \\
a_1 & 1 & \ldots & 0 & b_2 & b_1 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
a_{n-1} & a_{n-2} & \ldots & a_1 & b_n & b_{n-1} & \ldots & b_1 \\
a_n & a_{n-1} & \ldots & a_2 & 0 & b_n & \ldots & b_2 \\
0 & a_n & \ldots & a_3 & 0 & 0 & \ldots & b_3 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & a_n & 0 & 0 & \ldots & b_n
\end{pmatrix}$$

with $\psi = \Sigma\, b_i\, s^{n-i}$, $\phi = s^n + \sum\limits_{i=1}^{n} a_i\, s^{n-i}$, $C = (\sigma_1 \cdots \sigma_{n-1}\ \zeta_1 \cdots \zeta_n)^T$, and

$D = (d_1 - a_1 \cdots d_n - a_n\ d_{n+1} \cdots d_{2n-1})^T$. The matrix $S(\phi, \psi)$ is nonsigular when ϕ, and ψ are coprime, and we have

$$C = S^{-1}(\phi, \psi)\, D \tag{21}$$

The closed-loop characteristic polynomial can further be parametrized by a set of stable poles as

$$\chi = (s + r)\prod_{i=1}^{n-1}[\,(s + g_i)^2 + h_i^2\,], \quad r, g_i > 0. \tag{22}$$

Therefore the vector D in (21) can be replaced by parameters r, g_i, and h_i. Using this parametrization the search for the solution of the equation (16) is always performed among stabilizing compensators.

V. Numerical Results and Conclusion.

Consider a beam control system with the rigid-body mode plus a vibrational mode. Its transfer function G(s) has poles and zeros as shown in Fig. 6. Notice the reverse order of the pole and zero. Results of synthesis with respect to two sets of (τ_1, τ_2) are shown in Table 1. With the stabilizing compensator parametrization the solution of (16) converges very well. We have compared the controllers obtained minimax approach with state-space optimal controllers. It is our experience that generally controllers obtained from the minimax approach have better robustness.

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[1] Martin, G.D., "On the control of flexible mechanical systems", Phd dissertation, Guidance and control laboratory, Stanford university, 1978.

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flexible experimental manipulator", the international journal of robotic research, the MIT press, vol 3, No. 3, fall1984.

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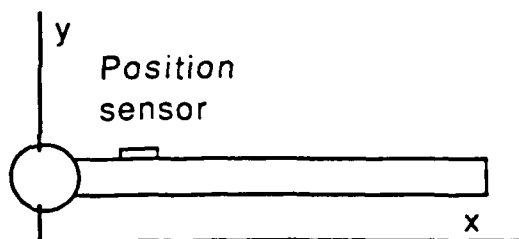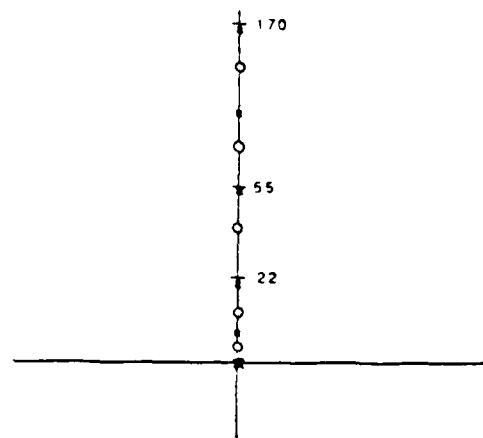
Figure 1. Beam control system.
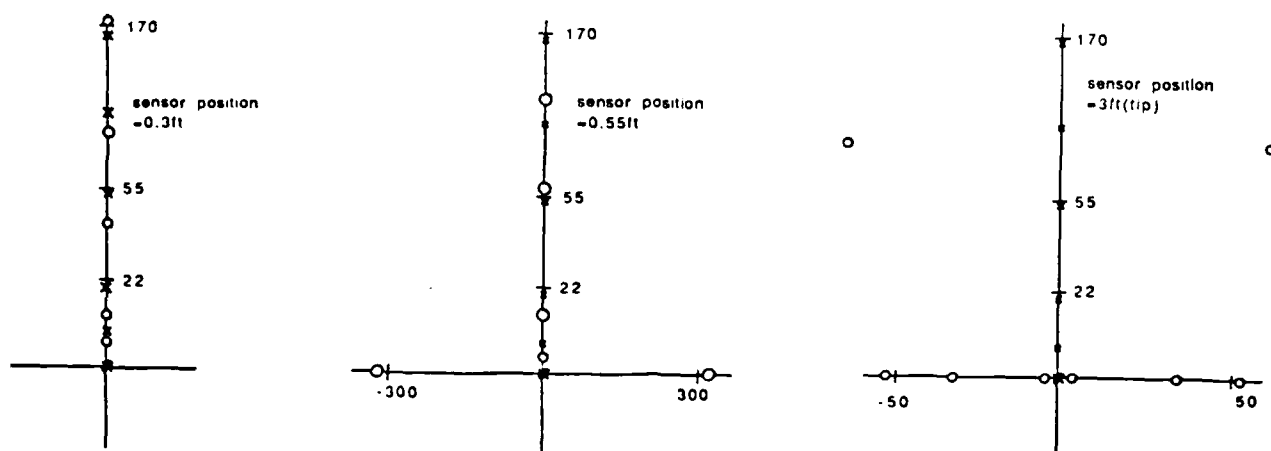

Figure 2. Pole-zero pattern of
the colocated system.
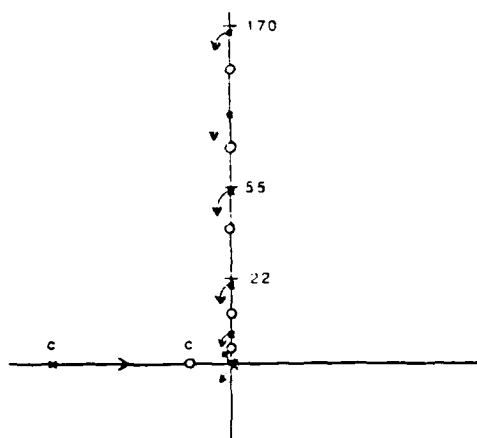

Figure 3. Zero pattern of the
noncolocated system.

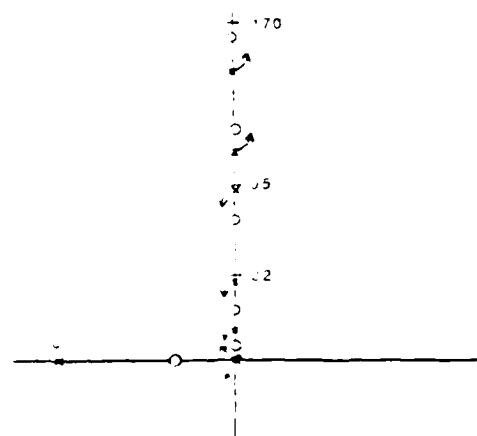Figure 4. Departure angles of the
colocated system with
lead compensation.

Figure 5. Departure angles of a lead
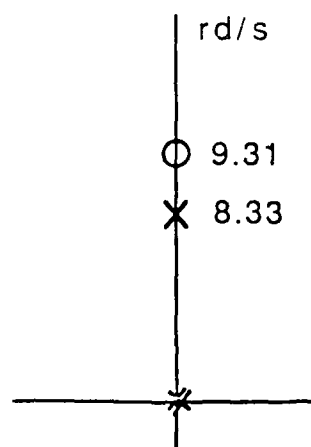compensated system with
poles and zeros reversed.

rd/s

○ 9.31

✗ 8.33

Figure 6. Design example.

$\tau 1 = 0.5$ $\tau 2 = 0.07$		$\tau 1 = 1.5$ $\tau 2 = 0.14$	
$\lambda = 0.1914$ $\sigma 1 = 21.417$	closed loop poles	$\lambda = 0.034$ $\sigma 1 = 24.083$	closed loop poles
$\sigma 2 = 416.554$ $\sigma 3 = -927.023$ $\xi 1 = -258.72$ $\xi 2 = 964.952$ $\xi 3 = 354.773$ $\xi 4 = 223.527$	 - 2 - 2 -5.41 -4.3+j3.3 -1.65+j5.2	$\sigma 2 = 552.07$ $\sigma 3 = -379.09$ $\xi 1 = -353.28$ $\xi 2 = 528.18$ $\xi 3 = 201.9$ $\xi 4 = 57.44$	 -0.67 -0.67 -7.54 -5.57+j3.24 -2.04+j5.62

Table 1. Data on $\tau 1, \tau 2$, H(s) and closed loop poles.

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